On foot navigation: continuous step calibration using both complementary recursive prediction and adaptive Kalman filtering

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1. INTRODUCTION

What is a more common displacement method for humans than walking? But behind this simple way of displacement are numerous complex physiological processes that are difficult to modelize. This complexity becomes obvious when one tries to apply classical GPS-INS algorithms (Britting 1971, Titterton 1997) to the data collected by miniaturized sensors carried by a person. If dead reckoning for vehicles can be satisfactorily resolved by means of inertial and map matching technologies, a similar approach is difficult to adopt for on foot navigation. The first problem to solve is the alignment of the IMU (Inertial Measurement Unit). Second, the inherent systematic errors present in small IMU quickly accumulate to non permissible positioning errors. Such characteristics do not allow one to compute his position by double integration of the acceleration. An alternative is to use accelerometer signal pattern rather than its value to count the steps (Judd 1997). Considering a constant stride, taking the pace count, and multiplying it by the step length provide the covered distance. As this parameter will mostly and directly influence the precision of the position during the dead reckoning phase, its parameterization is of first interest (Gabaglio 1999, Ladetto et al. 1999).

Stride detection in a continuously recording process is based on the assumption that one is able to identify when a step occurs. Depending on the accelerometer used and the sampling frequency of the signal, different strategies can be applied. Considering the Shannon theorem and the frequency of human motions, investigation are carried out using 40 Hz accelerometry raw data. This allows a precise timing of the step occurrence, as well as detailed pattern recognition.

The accuracy of the continuous step calibration will depend directly on the accuracy of kinematic positioning using differential GPS (Leick 1994). Within the range of variation of the strides, the ideally required precision is at the centimeter level. Such demanding accuracy is commonly reached after determining the cycle ambiguities (biases) of the carrier-phase observable for each satellite. Within the context of the present application, phase positioning is not reasonably conceivable. Several tests have shown that for short distances (up to 5 km) between the reference and the mobile receiver (i.e. the person), the 3-D positions calculated with the phase and with the code data match within 5 cm (Perrin 1999). The reason is that stride calibration needs only precise relative distance between two points rather than absolute localization.

The variation of a stride can be modeled as a function of several parameters: the step frequency, the signal variance and the incline of the road. For a realistic step length...
prediction, it is necessary to consider the internal, natural variability of the stride at a given frequency. As briefly discussed here, it is important to keep in mind that human stride is everything but constant.

It is obvious the traveled distance without continuous heading information is not relevant. In order to provide all necessary parameters once navigating without satellites, we are integrating a GPS receiver to a high accuracy positioning module (Leica DMC-SX™). This device includes three magnetic field sensors, three accelerometers which are also used as tilt sensors, and a thermometer. All sensors are connected to an integrated data logger. This paper focuses on the step lengths determination. It presents a physiological approach that takes into account the individual natural stride variation of people, as well as different walk dynamics.

**Figure 1:** Accelerometry signal variance during different frequencies of walk.

The structure of the paper is as follows. First, a description of the accelerometer signal as well as the analyzed step pattern is given. Then the step detection procedure is presented together with its modelization and a wavelet pre-processing of the signal. Finally, the dead reckoning calibration procedure using recursive prediction and adaptive Kalman filtering is discussed. Closed form formulas characterizing adaptive Kalman filter are presented, and the results are shown for several outdoor tests.

### 2. DISPLACEMENT DETECTION

Accelerometry is a very powerful tool to modelize different kinds of human activities (Aminian et al. 1999). The interest here is not to know if a person was standing or sitting, but only to detect when that person is walking. Using tri-axial accelerometers, intuition guides us to analyse the vertical and antero-posterior (direction of the run) signals. The computation of the signal variance at one second intervals permits the detection of walking periods. The mean variance over a short period appears to be well correlated with the step frequency. The Figure 1 displays the computed variance of the vertical signal during walking at different given frequencies. An individual calibration is necessary to fix the correct walking threshold, especially when people move very slowly. Effectively, a small variance value for someone who walks very lightly will not imply a displacement for one who strongly beats the steps.

Different tests (Perrin et al. 2000) have shown that the most natural and pleasant step frequency seems to be around 110 steps/min (~1.8Hz). This will play a significant role during the calibration.

### 3. STEP DETECTION

All the necessary information to detect a step occurrence is found in accelerometer signal. Using the same example, detection algorithms can be applied on both the vertical and antero-posterior signal. Several identification strategies are possible, but we show the one that appears to give the most robust results with the least computation time. The global idea is to localize maxima within a fixed interval. The size of this interval depends on the analyzed signal. When working with vertical acceleration, a characteristical pattern of two peaks, close in time, can appear at each step depending where the sensors are placed on the body. These correspond to the impacts of the heel and of the sole with the ground. The heel impact normally shows the biggest value for flat and light incline walks, but the pattern also varies from one person to the other. Mechanics completely changes once the slope is becoming larger than 10%.

**Figure 2:** Typically shifted antero-posterior and vertical acceleration pattern while walking. Sensors are placed on the low back (up) or on the thorax (low).

The antero-posterior acceleration presents one main maximum, corresponding to the heel impact, as can be seen in Figure 2. Physically, this represents the forward
displacement of the body. Ideally integrating this signal twice, after the attitude determination of the IMU, should permit to deduce the step length. The step identification using the presence of both shifted peaks should be considered as the most physiologically correct strategy. The rapid and brief variation of both individual accelerations allows to work with only one signal to give a robust step detection. A combined analysis has been tested, and validated the chosen approach.

As one step will be defined as the traveled distance between two heel impacts, this introduces a necessary notion of time interval between them. If a maximum is not followed, after a certain time, by another one, the person is still considered at the previous location. Such singularities generally occur during short and non regular walking periods.

Taking wrong time intervals will give an over(-under) evaluated number of steps. In dead reckoning mode, this rapidly leads to errors of tens of meter in long traveled distances. Such an error source can be partially removed by band-pass filtering the signal or by applying the wavelet transformation (Matlab 1998, Thonet et al 1998). A wavelet transform is used in the following.

The original signal passes through two complementary filters. It is divided into its low frequencies (approximation) and high frequencies (detail) components. By iterating this process $n$ times on the resulting approximation, the signal is broken into lower resolution components. This is called the wavelet decomposition tree, and $n$ is the number of computed levels. The choice of the wavelet family is function of the signal characteristics to analyze. In the present application, the pattern of the acceleration is lost to the benefit of a better shape. Figure 3 presents both raw and pre-processed data using the Meyer wavelet function. The signal decomposition was performed at level 4. The detail at this level reproduces the step frequency very well, with one maxima only at each occurrence.

![Figure 3: Raw and pre-processed signal using Meyer wavelet function at level 4 decomposition.](image)

4. STEP MODELIZATION

Ideal modelization is achieved when the measurement totally fits the observable through the model. If humans walked with constant steps length, the calibration would be easy. But as beautifully expressed by the English poet Abraham Cowley : "The world is a scene of changes and to be constant in Nature were inconstancy!". Several outdoor tests have demonstrated different walking particularities.

- Certain people show an “ideal” step length, independent of the frequency and of the road incline. Athletic people often present this kind of particularity.
- There is not always symmetry between steps of right and left legs. Step lengths are rarely equal, and 5 cm differences are common.
- Globally, a good correlation ($>0.6$) is observed between the step length and the step frequency.

Taking into account all precedent observations, the predicted step length will be computed using the following equation:

$$\text{Step length} = A + B \cdot \text{Freq} + C \cdot \text{Var} + w \quad (1)$$

$A, B, C$ : Computed parameters by linear regression

Freq : Actual step frequency

Var : Variance of the signal

$w$ : Gaussian noise $N~(0, \sigma)$

The step frequency can be determined with a changing number of occurrences using a Fast Fourier Transformation (FFT) or by time differencing the maxima. Since the dynamic of walk can change very rapidly, the smaller the calibration period, the quicker the adaptation of the estimated value. The estimation quality will then directly depend of the “individually” computed parameters of the regression. Once determined, they are fixed per person for the interval inside which the step sizes are varying. The continuous interval variation and recalibration is realized through the use of the adaptive Kalman filter.

5. ADAPTIVE KALMAN FILTERING

As currently defined (Kalman 1960, Brown & Hwang 1997), the Kalman filter is an optimal combination between the time propagated estimate from a previous time instant, and the measurement at the present instant. This optimal combination is dependent on the error variance of both the prior estimate and the current measurement. The Kalman filter estimates the state of a
dynamic system driven by white noise. The system, in the state space form is described by:

\[ x(k) = \phi(k-1) \cdot x(k-1) + u(k-1) \]  

(2)

\[ y(k) = H(k) \cdot x(k) + n(k) \]  

(3)

where \( x \) is the state vector composed of the variables that completely describe the dynamic system (steps length), and \( y \) is the observation vector of cumulated distances. \( u(k) \) and \( n(k) \) are uncorrelated and represent the process and measurement noise with known covariance matrix \( Q_d(k) \) and \( R(k) \), respectively. \( \phi(k) \) is the state transition matrix, and \( H(k) \) the observation matrix.

The Kalman filter update is a recursive two-step process. The time measurement update is given by:

\[ \hat{x}(k) = \hat{x}^-(k) + K(k) \cdot (\hat{y}(k) - \hat{y}(k)) \]  

(4)

where \( \hat{x}^-(k) \) denotes the calculated a-priori estimate. The “hat” denotes an estimate and the superscript minus indicates that this is the best estimate prior to assimilating the measurement at time \( t_k \).

\[ K(k) \text{ if the gain calculation and can be written as:} \]

\[ K(k) = P^-(k) \cdot H^T(k) \cdot [R(k) + H(k) \cdot P(k) \cdot H^T(k)]^{-1} \]  

(5)

where \( P \) is the error covariance matrix, computed for stability reasons (Farell & Barth 1998, Merminod 1989) in the Joseph Form:

\[ P(k) = [1 - K(h) \cdot H(k)] P(k) [1 - K(h) \cdot H(k)]^T + K(k) \cdot R(k) \cdot K(k)^T \]  

(6)

The recursive Kalman filter gain can then be extrapolated in time as follows:

\[ \hat{x}^-(k) = \phi(k-1) \cdot \hat{x}(k-1) \]  

(7)

\[ P^-(k) = \phi(k-1) \cdot P(k-1) \cdot \phi(k-1)^T + Q_d(k-1) \]  

(8)

The adaptive context comes from the processing noise uncertainty and variability. In this application, no standard values are available. The most probable value comes from examining the physics of the problem. \( Q_d \) represents here the uncertainty by which the predicted step length can match the true value. As filtered steps values are supposed constant for a definite interval, the bigger the residuals with the predicted steps length (1), the bigger the \( Q_d \) value. Computing the Gaussian distribution of this residuals will give an information about the processing noise.

6. DESIGN OF THE COMBINED FILTER

The step length will be predicted using a recursive least squares approach when GPS data are not available. The number of steps taken into account to predict the next value will influence the time response of the filter to an abrupt change in the step length (e.g. walk \( \rightarrow \) run). The different tests were conducted with a 20 steps update period. All studies made on an average of 20 people brought to the fore that the step length is more irregular when walking slowly. Values might vary from 4% at 130 steps/min rate walk to 15% for a 60 steps/min walk. If we consider a mean step value of 75 cm, the standard deviation of the step length varies from 3 cm to 11 cm depending on the frequency.

![Figure 4: Predicted step values at different frequencies with normal distribution of the error.](image)

Taking into account this natural characteristic, a following prediction procedure is adopted:

1. Consider a constant step value \( S \) on the interval of \( t \) steps
2. Compute the residuals between \( S \) and the predicted value using equation (1)
3. Estimate the mean and the variance of the normal distribution of these residuals \( N_{\sim}({\mu}, {\sigma}) \).
4. Compute the next constant \( S = S + \mu \)

This approach takes into account the natural behaviour of human walk. Although steps are not constant, they are normally varying around a more stable value. The presented procedure tries to take advantage of this property considering the Gaussian noise distribution. At the same time, it smoothes the effect of possible outlier values that can happen with singular individual accelerometer measurement. Figure 4 displays the predicted step values at different frequencies, showing the internal variance of the steps. As mentioned earlier, the biggest variance occurs at the lowest frequency.

When GPS data are available, they will permit both a recalibration of the step length and the computation of the regression parameters of equation (1). The state space of the adaptive Kalman filter is then:
step(k) = step(k-1) + u(k-1) \hspace{1cm} (9)

Distance (GPS)/# of steps = \text{step}(k) + n(k) \hspace{1cm} (10)

Both noises are assumed to be Gaussian. The measurement noise is fixed to N~(0, 5 [cm]), and the process noise is initialized with N~(0, 10 [cm]). The state matrix is fixed to the identity and the observation matrix simply equal to 1.

The implementation procedure is the following:

1. Measure the stride length given by the GPS distance divided by the number of steps deduced from accelerometry (10).
2. Compute the stride length (S) coming from the prediction procedure.
3. Apply the Kalman filter (4)-(8).
4. Compute the \( \tau \) residuals between the linear prediction (1) and the new filtered value and estimate the Gaussian distribution parameters N~(\( \mu \), \( \sigma \)).
5. Consider \( \sigma^2 \) to compute the new “processing noise” value in (8).
6. Update the regression parameters in function of the observed step frequency.

If GPS measurements occur at one walking frequency only, the update is done only on the \( \Lambda \) parameter of (1). It corresponds to the average step value. Other parameters are kept to the previous values until new frequencies can be observed.

The adaptive Kalman filter supply an adaptation of the model to a changing walking dynamic of the person. Figure 5 presents both the recursive prediction and the adaptive Kalman filtered value of steps length after a change in walking dynamic. The Kalman filter allows a complementary and quicker re-parameterization of the predictive parameters, once GPS data are available. The precision we get regarding the prediction depends directly on the type of satellite data available, code or phase.

\[
\begin{align*}
\text{True step length} & \quad \begin{cases} 
\text{Recursive Prediction only} & \text{Adapted Kalman filtered values} 
\end{cases} 
\end{align*}
\]

\textbf{Figure 5:} Necessity to re-parameterize the predictive equation by mean of adaptive Kalman filtering.

In order to validate the approach described in this paper, studies were done with data from 22 different subjects. The errors on the traveled distances vary from 0.45% to 1.93%, and this, independently of the distance considered. Courses were realized on standard asphalt roads with slopes varying up to 17% and on tartan athletic tracks.

\textbf{SUMMARY AND CONCLUSION}

While almost solved for vehicle navigation using odometers and map-matching, dead reckoning for on-foot applications requires another and more individual approach. The accelerometry signal is not integrated to deduce the position, but used to localize the steps occurrences. As wheel circumference for cars, the stride length is the fundamental parameter for pedestrian dead reckoning strategies. More dependent on individual walking characteristics than on accelerometer errors, nature imposes its own modelization limits from which we can cautiously derive the following rules:

- As steps length is not constant but exhibits a continuous variation around a more stable value, the Gaussian approximation seems the most appropriate model. This concretely means that under-estimated steps length are compensated by over-estimated ones when computing the distance traveled.
- The analyzed tests of several walking frequencies show differences between the effective and predicted distance of less than 2%. For example, this results into a difference of 40.8 m for 2'300 m distance realized in 2'905 steps. In other words, this corresponds to a distributed error of 1.4 cm per step. Such value is fully acceptable in view of the previous remark.
- The tests reveal a non significant impact for slope smaller than 10 %. As the studies were made with young and healthy people, this remark is to be wisely considered.
- Special conditions such as stair walking, as well as displacement in more “unfriendly” pedestrian regions, still have to be studied to refine the modelization. The influence of ground structures on the walking dynamic is also to be considered.

Facing the wonderful complexity of human beings, any adapted step modelization cannot be realized without a basic understanding of its physiological aspect. This is the motivation behind such multidisciplinary approach.

\textbf{ACKNOWLEDGMENTS}

I would like to specially thank professor Bertrand Merminod and Dr. Jan Skaloud for their informed advices. Thanks also to Mr. Philippe Terrier and Dr. Yves Schutz to introduce me to the world of applied physiology.
The results presented here are based on studies performed under a project supported by the Swiss National Science Foundation.

REFERENCES


